Physics 200-05

Assignment 8

1. Consider $H = \frac{1}{2}\vec{h}\cdot\vec{\sigma}$ as the energy matrix. and that $\vec{n}_\psi\cdot\vec{\sigma}$ with $\vec{n}_\psi\cdot\vec{n}_\psi = 1$, is the matrix for which $|\psi\rangle$ is the eigenvector for the eigenvalue $+1$. It can be shown that

$$\vec{n}_\psi = \langle\psi|\vec{\sigma}|\psi\rangle \tag{1}$$

Show that the equation of motion for $\vec{n}_\psi$ is given by

$$\frac{d\vec{n}}{dt} = \frac{1}{\hbar}\vec{h}\times\vec{n} \tag{2}$$

Let us assume that initially $n_{\psi 3} = 1$ and $n_{\psi 1} = n_{\psi 2} = 0$. and that $h_3 = \epsilon\cos(\theta)$, $h_1 = \epsilon\sin(\theta)$, $h_2 = 0$.

Then as a function of time show that

$$\begin{aligned} n_3 &= (\cos(\theta)^2) + \cos(\omega t)\sin(\theta)^2 & (3)\\ n_1 &= \sin(\theta)\cos(\theta)(1-\cos(\omega t)) & (4)\\ n_2 &= -\sin(\theta)\sin(\omega t) & (5) \end{aligned}$$

where $\omega = \frac{1}{\hbar}\epsilon$.

σ_3 is the attribute which represents the location of the N atom in ammonia, the probability of tunneling from the right ($+1$) to the left side (-1) becomes small as the difference in expectation value of energy for the particle on the left or right differs by more than the "tunneling energy" ($\epsilon sin(\theta)$).

Show that if $\theta = \frac{\pi}{2}$, there is a complete tranfer of probability from the right to left (+1 to -1 eigenvalue). As θ goes to zero, the probability of finding the particle on the left at any time goes to zero.

(Note that this is an experiment being done by Walter Hardy and his group in the basement of Hennings right now, where instead of N in ammonia is the orientation of a giant molecule containing iron. Their big problem is to ensure that the angle θ is as near $\pi/2$ as possible)

2. Show that an alternative way of solving the time dependent equations of motion of the two level system is by directly solving

$$\frac{d|\psi\rangle}{dt} = -\frac{i}{\hbar}H|\psi\rangle \tag{6}$$

Assume that $H = \frac{\epsilon}{2}\sigma_2$, and that $|\psi\rangle = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}$ with $\psi_1(0) = 1$ and $\psi_2(0) = 0$. Find ψ_1 and ψ_2 as a function of time.

3. Consider the state for the two two-level states given by

$$|\psi\rangle = N_\psi \left(|+1;1\rangle \otimes |-1;1\rangle + |-1;1\rangle \otimes |+1;1\rangle\right) \tag{7}$$

$$|\phi\rangle = N_\phi \left(|+1;1\rangle \otimes |+1;1\rangle + |-1;1\rangle \otimes |-1;1\rangle\right) \tag{8}$$

where $|+1;1\rangle$ means the eigenstate with the $+1$ eigenvalue for the attribute Σ_1 in the case of the first particle and $+1$ for Ξ_1 for the second. Ie, the first value is the eigenvector, and the second is the sigma matrix of which this is the eigenvector. Σ_i are the three sigma matrices for the first particle, and Ξ_i are the three sigma matrices for the second particle. What are possible values for the normalisation factors N_ψ and N_ϕ? What is the inner product $\langle\phi||\psi\rangle$.

(Do not try to expand out the direct products in terms of matrices.)

4. If Σ_1 is the Pauli spin matrix for the first particle with matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and Ξ_1 is the Pauli spin matrix for the second particle with the same matric $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, show in problem 3 that both $|\psi\rangle$ and $|\phi\rangle$ are eigenvectors of the matrix $\Sigma_1 \otimes \Xi_1$. What are the eigenvalues?

5. Show expicitly that if

$$|1;3\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{9}$$

$$|-1;3\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{10}$$

for each of the two particles, then the four dimensional vector

$$|1;3\rangle \otimes |1;3\rangle + |-1;3\rangle \otimes |-1;3\rangle \tag{11}$$

cannot be writen as a simple product

$$|\chi\rangle \otimes |\xi\rangle \tag{12}$$

for any choice of $|\chi\rangle$ and $|\xi\rangle$. Such a vector for two particles which cannot be written as a simple product of vectors for the two single particles is called an entangled state.

(In this case do expand the direct products of ket vectors in terms of matrices.)

[Note: in the physics literature, that $\otimes$ symbol is almost always omitted. Thus $|\chi\rangle \otimes |\xi\rangle$ is written as $|\chi\rangle|\xi\rangle$ and you are expected to know that it is the direct product that is being used if you are refering to two separate particles. Similarly $\Sigma_1 \otimes \Xi_1$ is written as $\Sigma_1\Xi_1$ where you are to remember that this is a direct product not a matrix product because Σ_1 and Ξ_1 belong to two separate particles. It is never correct to matrix multiply attributes which belong to separate particles.]