Aharonov Bohm

I. MAGNETIC FIELD FREE GAUGE

Consider a region of space which is free of magnetic fields but contains a non-zero vector potential field. An example of this is the region outside an infinite solenoid with an axial current running through the walls of the solenoid. (anotehr eample would be a cylindrical magnet the ends of the magnet being connected to each other by a massive piece of mu-metal which conducts the field from one end to the other without "any" (well very small) magnetic field in the space around the magnet.

For an infinite solenoid the magnetic field inside is determined by the current running thorught walls, while the field outside is zero. A possible vector potential will be a vector potential which runs in circles perpendicular to the solenoid.

$$B_x = B_y = 0; \quad B * z = B0\Theta(R-r) \tag{1}$$

$$A_x = \frac{1}{2}(B0y\Theta(R-r) + \frac{R^2}{r^2}B0y\Theta(r-R)) \tag{2}$$

$$A_y = -\frac{1}{2}(B0x\Theta(R-r) + \frac{R^2}{r^2}B0x\Theta(r-R)) \tag{3}$$

where $r^2 = x^2 + y^2$. The curl of $\vec{A}$ is 0 for $r > R$ and equals $B_z = B0$ for $r < R$.

Now take a point outside $r = R$, and connect this to another point outside $r > R$ and choose two arbitrary paths from the first point to the second point, such that one can deform one of the paths to the other without ever entering the region $r < R$. Ie, both paths run through the region $r > R$ and the inside of that path (the surface obtained in your deformation of the one path to the other).call these paths P1 and P2. Call the inverse of these paths (ie running from the second point to the first) $-P1$ and $-P2$. Then Stoke's thm tells us that

$$\int_{P1} \vec{A}_i \cdot dx^i + \int_{P1} A_i dx^i + \int_{-P2} A_i dx^i \tag{4}$$

is a closed path, and thus the integral around the closed path equal the surface integral of $\epsilon^{ijk}\partial_j A_k n_k d^2x$ equal the above closed integral. Sine $B = 0$ everywhere on this surface, both intgrals

$$\int_{P1} \vec{A}_i \cdot dx^i = \int_{P2} \vec{A}_i \cdot dx^i \tag{5}$$

Defining $\Psi(pf) = \int_{P1} \vec{A}_i \cdot dx^i$ which is the independent of the path we choose to get from the first point to second where p_f is the final point.

Then

$$\tilde{A}_i = A_i - \partial_i \Psi(pf(x^i)) = 0. \tag{6}$$

The change from A_i to $\tilde{A}_i$ is a guage transformation. Ie, we can always find a gauge transformation whith goes from A_i to 0.

However, there is another gauge transformation. Instead of going from the first point to the second, one chooses another path goes around the other side of the solenoid. as the first path above, such that the deforming the this path to the first path cuts through the region $r < R$ once. Lets call this $P3$. Then the integral

$$\int_{P1} A_i dx^i + \int_{-P3} A_i dx^i = \int_{S13} B_i n^i d^2S = B0\pi R^2 \tag{7}$$

There of course still exist an infinite number of paths whose surface does not go through $r < R$, and they can be used to define another gauge transformation $\hat{\Psi}$ which again set $\hat{\tilde{A}}_i = 0$. But on points where both Ψ and $\hat{\Psi}$ are defined, they differ by $B_0\pi R^2$. Ie, although in both cases the gauge transformed A is zero, the gauge transfomation itself is multivalued. One could find other gauge transfomation where for any integer N, positive or negative, we would have

$$\hat{\tilde{(x)}}\Psi - \tilde{\Psi} = NB0\pi R^2 \tag{8}$$

II. AB EFFECT

Now consider a complex valued null scalar field where I have chosen units of the magnetic field such that $\mu_0 = \epsilon_0 = c = 1$.

$$\partial_t^2\phi - \nabla^2\phi = 0. \tag{9}$$

We can give this field a charge, and minimally couple this field to the electromagnetic field buy the process

$$(\partial_t + iq\Phi)(\partial_r - iq\Phi)\phi - (\nabla_j - eA_j))(\nabla^j - A^j)\phi = 0. \tag{10}$$

where Φ is the electric potential and A_i is the magnetic potential. q is a constant of nature which converts the units of Φ and A_i to units of inverse time and space (since $c = 1$ time and space have the same units). That this consant q in the world we live in has units of $C/\hbar$ where C is the units of charge and $\hbar$ is Planks constant does not mean that there is anything quantum mechanical about this field ψ. It is a classical field. In out system we have no electric field, and thus have no potential Φ. The field A_i is taken to be time independent.

The charge density corresponding to thie field is

$$\rho = Im(iq\phi^*\partial_t + iq\Phi\phi)) \tag{11}$$
$$J^i = Im(iq\phi^*(\partial_i - iqA_i)\phi) \tag{12}$$

Now let us assume that the field A_i is a continuous single valued field, such as the field that we looked at at the beginning of this note. We will assume that the solution of the equations of motion for ϕ is single valued. We now carryout a gauge transformation on the field ψ such that $\tilde{\psi} = e^{+i\Psi}\psi$ (recall that Ψ is independent og time.) Then the equations of motion become

$$0 = \partial_t^2\tilde{\psi} - (\nabla_i - iqA_i)(\nabla^i - iqA^i)\tilde{\phi} \tag{13}$$
$$= e^{i\Psi}i\left((\partial_t^2\tilde{\psi} - (\nabla_i - iqA_i + i\partial_i\Psi)(\nabla^i - iqA^i + i\partial_i\Psi)\phi\right) \tag{14}$$
$$= e^{i\Psi}i\left((\partial_t^2\tilde{\psi} - (\nabla_i - iq\tilde{A}_i)(\nabla^i - iq\tilde{A}^i)\phi\right) \tag{15}$$
$$= e^{i\Psi}(\partial_t^2 - (\partial_i\partial^i)\psi = 0 \tag{16}$$

since $\tilde{A}_i = 0$. If we take a wide enough laser beam of frequency ω and split it using a beam splitter The solution along a straight path will could be something like

$$\phi = e^{-i\omega(t-\vec{n}\lambda}e^{\frac{-s^2}{\sigma^2(\lambda)}} \tag{17}$$

where for large σ it will change slowly with λ. $\vec{n}$ is the direction of travel of the beam, and $lambda$ is the distance along the path of the beam.

We now create the beam with a beamsplitter, bounce the two beams off mirrors so that they take different paths around the beamsplitter, and then interfere them through a second beamsplitter. We take the common initial point of the beam at the first beansplitter, and recombine the beams at the second beamsplitter. While in the first path 1-2-4, we chose the Ψ as the gauge transformation to make the vector potential zero, for the second path 1-3-4 we choose $\hat{\Psi}$ as the gauge transformation. Once we get to teh secon (4) beamsplitter the phase of teh solution in the original gauge will be $\omega(\lambda_4 - \lambda_2 + \lambda_2 - \lambda_1)$ on the second path we will have $\omega(\lambda_4 - \lambda_3 + \lambda_3 - \lambda_1) + B_0\pi R^2$. The difference between these two phases will determine in which direction the beam will exit the second beamsplitter. If the phase is $2n\pi$,it will exit one way, while if it is $(2n + 1)\pi$, it will exit the other way. Thus if we leave the interferometer alone and we change the amplitude of the magnetic field, we can tune the phase difference to whatever multiple of π we want.

We note that the beam always travels through a region where the B field is 0. The B field cannot directly affect the charged beam. Furthermore the A field is gauge equivalent to 0 at each point along the path, so that again shoul not effect the beam. And yet, as we change the B field inside the solenoid, the interference pattern at the beamsplitter 4 changes. Ie, sustem's response depends on which gauge transformation we carry out. The gauge is physical, or rather

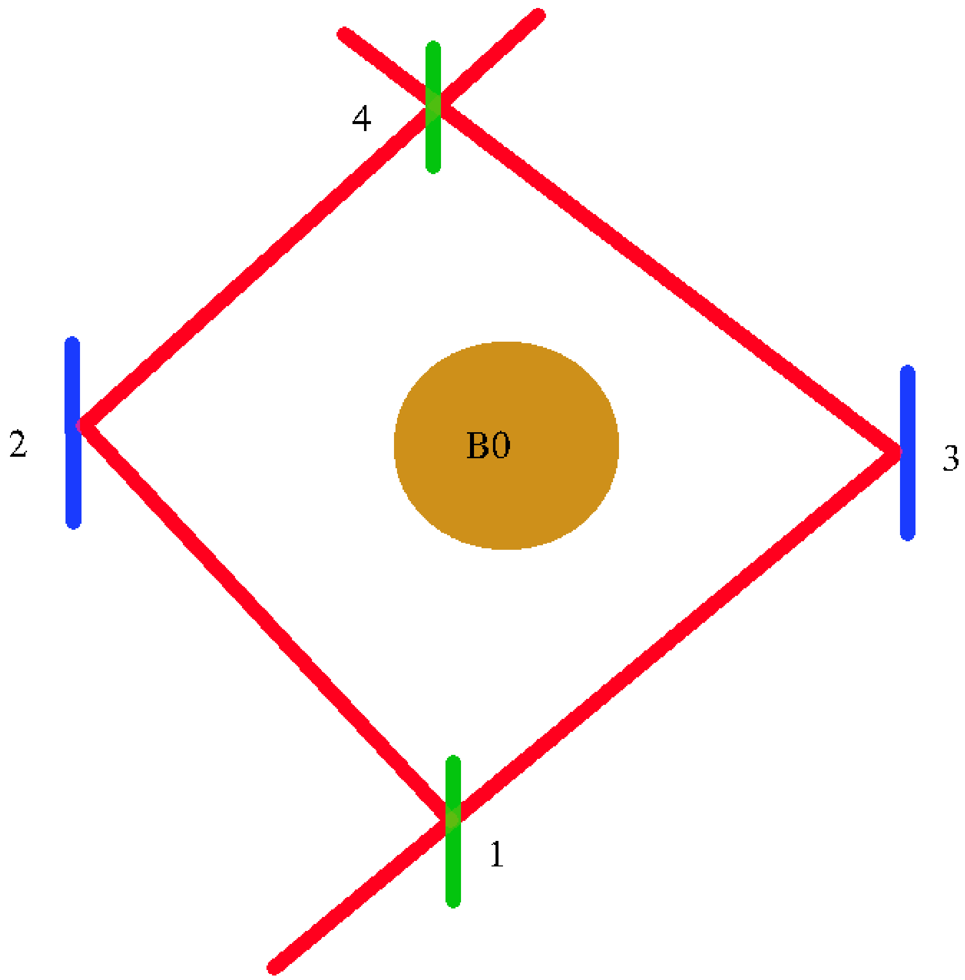

FIG. 1: Figure AB-interf. AB interferometer

gauge invariant aspects of the gauge transformation are phyical, including the path intgral of the gauge around a closed loop is physical.

This came as a shock to everyone, and many refused to believe it, until finally experiments were carried out which demonstrated the effect.

It is important to note that while I used a field ϕ which obeyed a massless field equation, one could equally use a field which had an addition term of the form $m^2\phi$, to make it a massless scalar field.

If one quantizes the field, the factor $q\hbar$ will be the charge of the "particles" associated with this field. But again the AB effect is a classical effect, which can be quantized.
