Physics 301-24
Assignment 2

1. Multiconnected Stokes Thm

a)Consider a surface with a hole in the middle. There are two ways of looking at this. Teh first is to put the "origin" of the coordinate system in the hole and to make the edge of the hole to lie at $x^2 = .5$ and the other outer edge to lie at $x^2 = 1$. The second is to make it into a single connected piece by cuting the surface in a line from the inner hole edge to the outer edge. Argue that the two give the same result. (Hint: what happens with the line integral from the cut surface along the cut edge(s)

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[2]There are a number of ways of doing this. i) fill in the hole in the surface. Now the integral of the perpendular length of the curl over the whole system gives the integral of the component parallel to the edge over the outer edge of the surface. The Then do the same calculation for the filled in part of the surface. This will give the integral over the inner filled in part of the perpendicular curl is equal to the integral over the edge of the filled in circle of the parallel pat of C. The integral of the surface with the hole is then the difference between the two, which is the differenc between the integral over the outdsice edge and inside edge.

ii) fill in the hole and place the reference point inside the filled in hole. Choose x^2 at the edge of the hole to be .5, and the outer 1. Then the integral of x^2 will go from .5 to 1, and the inner one will not be 0, as happened in the proof I gave. Instead we will have the difference between the outter integral and inner., as above

iii) Place a cut from the outer edge to the inner one. The resultaint piece will be simply connected (n holes) Carry out the procedure as in class on the single piece. The will be the itegal over the outter edge plus the integral down the cut from the outter to inner, edge, around the inner edge in the opposite direction, and then along the cut from inner to outder. Teh contribution from the two parts of the cut will be equal and opposite and thus cancel. So the integral will around the outter edge in one way plus the integral around the inner edge in the opposite direction. But an integral in the opposite direction is negative that in the same direction, so again we get the difference betwen the outer and inner edges.

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2.) Gauss's law and Stokes law.

i)In a conductor, there can be no static electric field, because an electric field will cause the charges to move around, until the electric field inside is 0. The moved around charges will be deposited in a thin layer on the surface of the conductor.

Using Gauss's law and Stoke's law, what will be the direction of the electric field be just outside the conductor.

ii) In a type 1 superconductor the magnetic field is zero inside the superconductor. What will the direction be of the magnetic field be just outside the superconductor. (In this case the currents will be moved to a thin layer on the surface and will be distributed so as to cancel the magnetic field inside.)

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[4]Inside the conductor there is no electric field, and outside there may be. There is a charge along the surface. of the conductor. Create a little surface which is close to the conductor surface outsice, then dives into the inside of the conductor, travels back hugging the surface from inside, ad then travells out. The integral over the interior is the divergence of E (which is the charge density over ϵ_0,, which is equal to the integral of the perpendular component of E over the outside surface plus that over the inside surface, plus a tiny little contribution over the sides.Teh inside integral is zero, the charge is non-zero, if there is outside electric fields. Thus the perpendicular component of the electric field to the outside surface (which since the surface hugs the conductor is also perpendicualr to the conductor). Ie, the outside electric field has a component perpendicular to the conductor.

Now use stokes them, and produce the same kind of thing. The curl of E equals zero in the static approximation. Create a path which hugs the inside of the surface crosses the conductor and then hugs the outer surface before it finally connect to the inside line inside the conductor. Thus the integral over the perpendicular component of curl E to the surface spanned by that line is 0, which means that the line integral is 0. Since inside the conductor, E is 0, the integral along the outside path is zero, no matter how short or long the path or what its direction is. Thus the component of E parallel to the surface is 0.

At a conductor the Electric field is perpendicular to the conductor. Since the E field is the gradient of the electric potential ϕ this means that the the derivative of the potential parallel to the surface is zero, which means that the potential must be constant along the surface.

For the magnetic field around a superconductor, you get the opposide. The divergence is always 0, so the gausses law integral over the surface will be zero, implying there cannot be a perpedicualr component of the magnetic field, while the curl is $\mu_0 J$ along the surface of the superconductor. Thus it is the curl equation which can be zero, inplying that the integral over that line give the contribution.

Thus E field come into conductors perpendicularly, and B field points along the superconductor just outside the supreconductor.Thus, placing a supercondutor over a magnet, compresses the magnetic field, and it is that compression that holds up the superconductor.

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3. [1]Consider an electric potential $\phi(x, y, z) = |x|$, What is the electric field everywhere? [1]Show that there is no charge anywhere except along the $x = 0$ surface. [2]What is the charge per unit area along the $x = 0$ surface. (Use the definition of the electric field as a function of ϕ? [1]What is the divergence of the electric field.

[The divergence of the equivalent of the electric field, the spatial derivative of the potential, would just be $(\partial_x \sqrt{g} g^{11} E_1)/sqrt(g)$ Since the metric for x in cartesian coordinates is $g_{11} = 1$. But $E = \partial_x \phi = \sigma(x)$ where $\sigma(x) = 1$ if x¿1 and is -1 if x¡1. Thus the divergence of E is 0 everywhere except x=0. Integrating the divergence from -1 to $-\epsilon$ and then from ϵ to 1, gives a 0 But the integral of the divergence integrated from -1 to 1 equals the difference in valued of E at 1 and -1. That differnce is 2. Thus the integral of the divergence over the whole interval is 2, but over -1 to $-\epsilon$ and then from ϵ to 1 is zero. This the integral from $-\epsilon$ to ϵ must be 2, no matter how small epsilon is. Thus the divergence of this E is zero everywhere except at 0, where its integral is 2. This is called distribution (something which makes no sense as a faunction, since for any function, the integral over a set of measure 0 is 0, not 2, but makes sense as an integral. This is usually called the Dirac delta funtion, even though it is not a function, and is written as $\delta(x)$ Someting whose integral over any region which does not include 0 gives 0, but if it includes 0 the integral is 1.

4. In a space free of charges, show that the potential cannot have a minimum or maximum.

[1]What is the condition that the potential have an extemum (minimum, maximum or saddle point).

[1]What is the condition on the second derivatives that the field have a maximum?

[1]Why can this not occur for a potential in free space. (Use Cartesian coordinates).

At a maximum or minimum, the derivative or the function is 0. So all three derivatives $\partial_x \phi = \partial_x \phi = \partial_z \phi = 0$ are zero.If itis a maximum however the second derivatives must however be negative. But the potential obeys

$$\partial_x^2 \phi + \partial_y^2 \phi + \partial_x^2 \phi = 0.$$

But you cannot add three negative numbers to get 0. Similarly for a minimum where all three second derivatives must be positive for a minimum.

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5. Consider a cylindrically symmetric electrical potential (ie, it is indepndant of z and θ where the metric is

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2 \tag{1}$$

[1]What is the equation for the potential. Assume that for r¿0 are no charges.

[1]What are the solutions for the potential equation given the above conditions.

[2]Using Gauss's law, what is the charge per unit length at $r = 0$ of the solutions inside a clinder of height $\delta z = 1m$ of your possible solutions.

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

and thus

$$g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

and $g = r^2$. Thus The divergence of the gradient is

$$\frac{1}{\sqrt{g}} \partial_i \sqrt{g} g^{ij} \phi(t, \theta, z) \tag{4}$$

$$= \frac{1}{r}\partial_r(r\ 1\ \partial_r\phi) + \frac{1}{r}\partial_\theta(r\frac{1}{r^2}\partial_\theta\phi + \frac{1}{r}\partial_z(r\ 1\phi) \tag{5}$$

$$\frac{1}{r}\partial_r(r\partial_r\phi(r)) + \frac{1}{r^2}\partial_\theta^2\phi(r) - \partial_z^2\phi(r) \tag{6}$$

$$\frac{1}{r}\partial_r(r\partial_r\phi(r)) = 0 \tag{7}$$

Then

$$\partial_r\phi(r) = \frac{C}{r}; \qquad \phi(r) = D + Cln(r) \tag{8}$$

Construct a cylinder ar r=1 with caps at $z = 0, \quad z = 1$ Construct another cylinder at $r = \epsilon$ (small)

Using Gauss's law, the integral isof the divergence of E is zero. Also the only component of DE is $E_r = \partial_r\phi(r) = \frac{C}{r}$ Thus the integral over the surface is

$$\int E_r r d\theta dz = \int \frac{C}{r} r d\theta dz_= 2\pi Dz$$

The surface integral is indepedent of r. no matter how small, which implies that there is a charge of $\epsilon 2\pi Dz$ located at r=0. or of charge per unit length of $\epsilon(2\pi D)$.

This is a 2 dimensional delta function. It is zero everywhere except at $r = 0$, but its integral over the $r\ \theta$ surface is non-zero.