Physics 301-24
Assignment 3

1.i) Show that the solution for the potential of a spherical layer of charge at $r = R$ with total charge Q is given by

$$\phi(r,\theta,\phi) = \begin{cases} \frac{Q}{4\pi\epsilon_0 r} & r > R \\ \frac{Q}{4\pi\epsilon_0 R} & r < R \end{cases} \tag{1}$$

Use spherical coordinates.

ii) The alternative would be to do the integral using the Green's function in Cartesian coordinates. Write down the integral you would have to do in order to get the potential everywhere. The integral is non-trivial and I do not expect you to do it.

iii)What is the Electric field inside $r = R$. (Note that in the gravitational force case, Newton had to go through muct argument to come up with this result in the gravitational case.)

iv) Translate the above to cartesian x, y, z coordinates, assuming that the origin of the spherical polar coordinates is at $x = 0,\ y = 0, z = 0$.

2.i) Assume we have two point charges with opposite $\pm Q$, the positive one centered at $x = 0,\ y = 0,\ z = a$, and the other at $x = 0,\ y = 0,\ z = -a$, Using Gauss's thm., or any other argument, what would the integral over a sphere of radius $\mathfrak{R} >> a$ centered at $x = 0,\ y = 0,\ z = 0$.

ii) What is the electric field along the x axis ($z = y = 0$) and along the z axis (x=y=0)?

iii) Far away from the charges ($r >> a$) what is the potential to lowest non-zero order in a/r?

3. i) Using the Green's function

$$G(\mathfrak{X}, \mathfrak{X}') = -\frac{1}{4\pi\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

which obeys

$$\nabla^2 G(\mathfrak{X}, \mathfrak{X}') = \delta(x-x')\delta(y-y')\delta(z-z')$$

find the potential for a line of charge, with charge density per unit length of σ $\rho(x, y, z) = \sigma\delta(x)\delta(y)\Theta(a-z)\Theta(z+a)$.

ii) What is the limit as $a \to \infty$. Compare this with the potential you derived in Assignment 2 for the potential for a cylindrically symmetric charge distribution.

iii) Do the same for the endpoints located at $z = 0$ and $z = a$

(Recall that $\Theta(z)$ is the step function, such that it is 1 for $z > 0$ and 0 for $z < 0$.)

Hint. The integral

$$\int_0^M \frac{1}{\sqrt{mu^2+\nu^2}}d\mu = -\ln(|\nu|) + \ln(M + \sqrt{M^2+\nu^2}) \tag{2}$$