Physics 301-24
Assignment 4

1. Consider a point in space containing an electric potential. Consider a sphere around that point of radius R and within that sphere the potential is source free. Show that the potential at the center of the sphere is the average of the potential over the surface of the sphere. (Hint define a Green's function $\tilde{G}(\mathfrak{X}, \mathfrak{X}')$ where $\mathfrak{X}'$ is the center of the sphere and $\mathfrak{X}$ is on the surface of the sphere. Find $\tilde{G}$ such that it is zero on the surface of the sphere. Use that to find the value of the potential at the center of the sphere in terms of the value of the potential on the surface.)

2) show that the monopole ($l = 0$) moment of a charge distribution is just the total charge, and the components of the dipole moment are given by

$$\int \rho(x,y,z)(x+iy)dx^3; \quad \int \rho(x,y,z)zd^3x; \quad \int \rho(x,y,z)(x-iy)d^3x \tag{1}$$

3) Consider a charge distribution with both a monopole ($l = 0$) and a dipole ($l = 1$ moment to the potential. Show that by changing the origin around which you calculate the spherical expansion, you can set the dipole moment to zero, but only if the monopole moment is not zero.

4) Consider the potential in cylindrical ccordinates with metric

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2 \tag{2}$$

Write the Poisson equation of this metric in the "seperation of variables" form.

Solve the angular and the z equation.

In the case that the solution is independent of z solve the radial equaiton.

In the case that the solution is not independent of z, the solutions of the radial equations are modified Bessel functions. Any second order differential equation has two indepdenent solutions. What are the behaviour of the two solution near $r = 0$. What are the behaviour of the two solutions near $r = \infty$. Note that in order to get regular solutions both at $r$near zero and r near infinity, one must have charges in the space. (This is another manifestation of the theorem tha tthat the Poisson equation without sources has only one regular solution, the potential is constant everywhere.

Note again that the seperation of variables works only if there are symmetries of the equation. In this case, translation of z and rotation around the z axis.
