Physics 301-24
Assignment 5

1).[9] (3xx) Using the bycycle wheel model of a magnetic dipole, one this time insert the wheel into a uniform magnetic field. (eg, inside a large and long cylindrical solenoid).

a)Consider the dipole axis oriented perpendicular to the B field. Show that there is torque (a force on the dipole which tends to bring the axis parallel to the B field.)

The loop is lying parallel to the $\vec{B}$ field which I will assume is along the z axis. We can therefor orient the loop as lying along x=0. Ie

$$\rho = \rho_0 delta(x)\delta(r-R) \tag{1}$$
$$J^x = 0 \tag{2}$$
$$J^y = -z\rho \tag{3}$$
$$J^z = x\rho \tag{4}$$

The force of the B field is $\vec{J}\times\vec{B}$. so the only force will be

$$Fx = J_y B_z = -z\rho(y,z)B_z \tag{5}$$

The torque is a force around the y axis. current in the y direction. Ie the force at the largest z=R points in the -x direction, while at z=-R, it points in the +z direction. The There is thus a torque on the current, that tries to align it with the axis along the z axis. This tilts the wheel by rotating it around the y axis, and thus produces an axial current around the y axis, procucing a current in opposite directions in the x direction. These currents cross the B field in the z direction, producd a $J\times B$ force in the y direction– ie and axial force whichis largest when z is largest on the circle of the wheel. This force is a braking force on the wheel spinning around the x axis, slowing down ω of the wheel. and extracting energy from the a=rotationof the wheel around the x axis. (see diagram)

Note that there is a subtily here. If the wheel has non-sero angular momentum, then applying a torque to the whee will cause it to precess, rather than simply tilting around the axis of the torque. However, this can be made not to occur by having an extra flywheel with opposite angular momentum to the wheel carrying the charge. that extra flywheel can be coupled to the charged wheel in such a way that the both always maintain the same angular momentum even though a braking is applied to only the charged wheel, but putting appropriate gears coupling the two wheels. Each wheel would carry angular momentum energy but the total angular momentum would be zero. (This begins to look like the gears and rotation that Maxwell postulated for giving an mechanical analog to the his electromagnetic field)

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b) Argue that the interaction energy in the electromagneic field around the dipole is a function of cos(θ) where θ is the angle between the axis of the dipole and the direction of the field, and that the maximum energy in the EM field is when the direciction of the dipole points in the same direction as the magnetic field. Ie, it will rotate around the y axis.

The electromagnetic energy is $\int \vec{J}\cdot\vec{A}$. For a constant B field, in the z direction the A field will be

$$\vec{A} = -\frac{1}{2}\vec{x}\times\vec{B} \tag{6}$$

since B is constant. We can check that this is correct by taking the curl of A

$$\vec{\nabla}\vec{A} = \frac{1}{2}\vec{\nabla}\times(\vec{x}\times\vec{B}) = -\frac{1}{2}(B\cdot\nabla(\vec{x})x - \vec{\nabla}\cdot\vec{x})\vec{B}) = \vec{B} \tag{7}$$

as required.
But

$$\int \vec{J}\cdot\vec{A}\mathfrak{V} = \int -\frac{1}{2}(\vec{J}\cdot(\vec{x}\times\vec{B})\mathfrak{V} = \int -\frac{1}{2}(\vec{B}\cdot(\vec{J}\times\vec{x})\mathfrak{V} = \int \vec{B}\cdot\vec{M} \tag{8}$$

where M is the dipole moment of the current. Since M starts off perpendicular to M, it starts off at 0. The torque on the current will try to orient M parallel to B. So the torque must go as $cos(theta)$

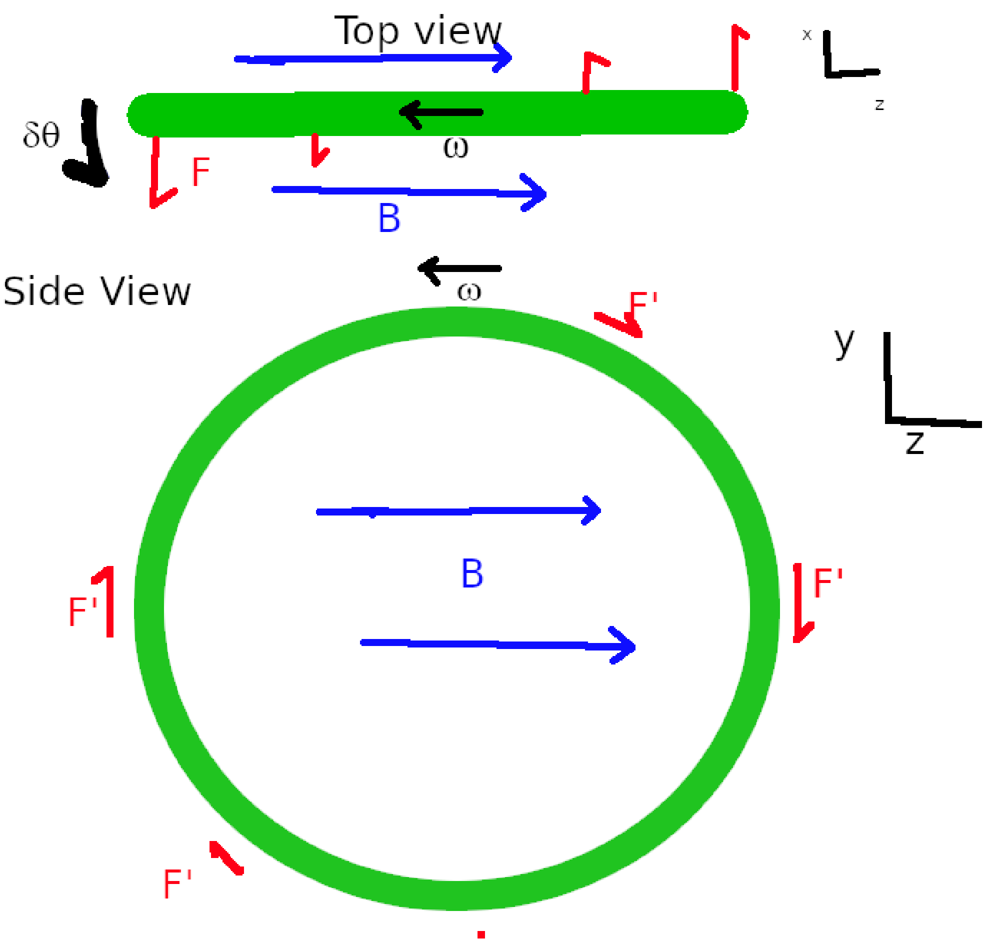

FIG. 1: Figure trans-wheel-forces. various forces on transverse wheel. J_y and B_z produce force in x direction of wheel, opposite directions on two sides of the wheel because current in opposite directions. That causes wheel to rotate around y axis, and the current that produces in x direction gives force on wheel tangential to the wheel and in opposite direction to J, which removes energy from rotation of wheel. To compensate for angular momentum of wheel (which causes precession) put another neutral wheel beside charged wheel with oppsite angualr momentum, and lined by geers to first wheel with opposite angular frequency and thus cancels the angular momentum of the charged wheel.

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c) Assume that you turn the wheel's axis slowly from pointing along the x axis to the z axis (along which the B field points) the work done by the torque (Torque time angle of rotation of the axis) must come from the kinetic energy of the rotating wheel. Where does the change in energy of the EM field come from? What happens to the mechanical energy of the wheel.

As the wheel's axis slowly rotates, the torque does work on the whatever it is that is slowly rotating the axis. But since the B field can do no work, the work done by the turning must come from the internal energy. Thus the energy which goes into the electromabnetic field MUST come from and E field. The turning wheel will create a time dependent B field and A field, and these create and E field. ($\partial_t B = -\nabla \times E$ or $E = -\partial_t A$) It therefore must be the reaction of the E field on the solenoid which creates the B field which supplies the energy needed to increase the energy of the EM field.

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2)[4] Previusly we found, from the stess tensor, that the both the E fieds and the B fields exert presures on the surrounding field. The pressure along the lines of the E field is outward, along the field (as though the lines of force were elestic bands) while the transverse pressue in directiond perpendicular to the field were an outward pressure, pushing the lines of force apart. Using these qualitative aspects use those to explain the form of the linesof force of the B field around a ring of moving charge.

The lines of the B field around the ring source, which look like the following picture, curve around the current loop, The loops spread out becaue of the pressure on the B field due to the off diagonal pressure pushing apart the B field lines. Howerver they do not expand out to infinity because of the tension in the lines of force (the force parallel to the B field which makes the lines act like rubber bands. So like a ballon, in which the internal tension of the rubber tries to make the baloon smaller, while the internal air pressure tries to make it larger. The compentition produces a rounded figure. The current loop provides a limit – the lines cannot be pushed into the current loop.The current sets the B field just inside and outside the current loop.

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3)[3] Consider a finite legth solenoid. The wire wrapped tangentially abount the cylinder has an internal resistance of R and has length L which is much greater than that circumference of the cylinder. a battery attached to .at time t=0 to the wire. What will you expect to happen to the B field and current as a function of time. As R goes to 0, what do you expect to happen to the current?

As the battery begins to run current through the solenoid, the B field and A fields start to change. Thus $\partial_t B = -\nabla \times E$ or $\vec{E} = -\partial_t \vec{A}$. The direction of this E field is in the opposite direction to the E field of the voltage of the battery $E = V/L$. The faster the current increases, the larger this bucking current is. This means that the E field in the wire is less, and the voltage, EL is smaler, driving less current throught the wire. That means that means that the rate of change of the current must be smaller through the wire is less than the maximum E/RL, and the current is less.

This backaction of the changing B field which is the proportional to the changing current is called the inductance of the solenoid, usuall designated by L (or let me call it $\mathcal{L}$ to distinguish it from the length.
