Physics 301-24
Assignment 7

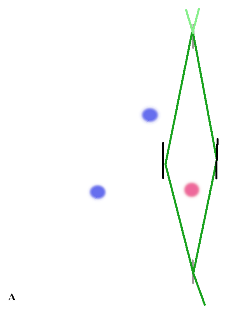
1) [8] Examine the diagram below of a system being used to detect the Aharonov Bohm effect. the red and blue circles are regions of the B field pointing into and out of the surface, the greed lines are the "paths" along which a beams of a scalar massive field are travelling, the solid black lines are mirrors for this fied, while the grey lines are 50-50 beamsplitters for the scalar field (ie the incoming bean splits into two beams with equal amplitudes in each outgoing beam). The original beam comes in from the bottom.

In which cases will a change in the strength B of the magnetic field in the "solenoids" change the ratio of the scalar field coming out of the two ports of the final beamsplitter? Why do you answer as you do?

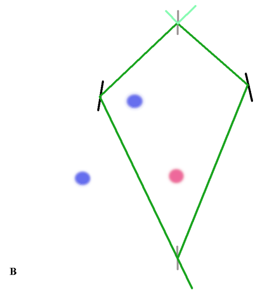
(Be careful with figure C)

The blue indicates a magnetic field of magnitude B pointing out of the page, while the red indicates a magnetic field of same magnitude B pointing into the page.

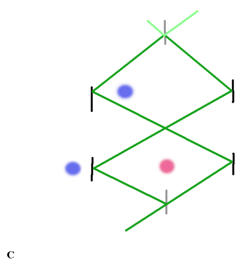
You may assume that the lenths of the paths from input to output of the scalar field are the same in each diagram



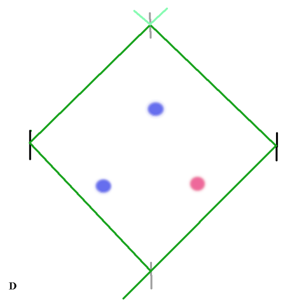
(a) A



(b) B



(c) C



(d) D

The ones where changing the value of B will change the relative amplitudes coming out the two sides of the top, output beamsplitter are A, C, and D. Changing the field does nothing in cases B.

The phase shift of the beams is due to two effects– the difference in path lengths of the two beams (which by assumption is zero) and the difference in the surface integral of the B field normal to the surface over the surface. In A, only the left blue one is circled by the two beams. In the second case, the beam circles two regions, one contributing the same value but opposite sign as the other. Thus the effect of the two B fields surrounded by the path cancels. In the third case, which looks similar to the second but is not. If we look at the path which starts going left on the bottom, minus the path which starts right at the bottom, it circles the B field in a clockwise direction for the bottom pink one, but circles the top one in a counterclockwise diretion. Teh normal to the surface spanned by the two parts of the path is opposite, since the B field is also opposite, that is two minus signs, so the contributions of the two field regions will add, rather than cancel. In D we have two regions where the B field is up, and one down, so the ups win.

Finally the B fields which lie outside the paths have no effect at all on the reduced A field inside the paths and have no effet on the AB effect.

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2.[4] Given the elecro- and magneto-statics equations

$$< E > (x) = \nabla < \phi_f > (x) \quad < B > (x) = \nabla \times < \vec{A} > \tag{1}$$

$$\nabla \cdot < \vec{D} > (x) =< \rho_f > (x) \quad \nabla \times \vec{<} H > (x) =< \vec{J}_f > (x) \tag{2}$$

For a linear medium, we also have

$$< \vec{D} > (x) := \epsilon(x) < \vec{E} > \quad < \vec{B} >:= \mu(x) < \vec{H} > (x) \tag{3}$$

$$< \vec{D} >= \epsilon_0 E + < vecP > \quad < \vec{H} >= \frac{1}{\mu_0} < \vec{B} > - < \vec{M} > \tag{4}$$

a) Show that the $\vec{D}_\perp$, the perpendicular component of D is continuous at a surface of discontinuous dielectric property, and the parallel component of E is continuous. (Use Gausses and Stokes thms)

We have $\nabla\times < \vec{E} >= 0$, so using Stokes thm on $< \vec{E} >$, with a narrow path following the surface and lying just outside an inside the surface of dicontinuity in ϵ. The integral of $\vec{E}$ along these paths equals the curl of $<\vec{E} >$ which is zero. Thus the integral along those two parallel paths must be equal. This says that the parallel components of E must be the same on both sides of the discontinuity.

On the other hand, if we make a Gaussian pillbox with the top and bottom of the pillbox parallel to the surface, the it is the integral of $< \vec{D} > \cdot\vec{n}$ along the top and bottom. that is zero since $\nabla \cdot D = 0$ and thus the volume integral of that diveregence is zero. Sincd the normal vector on the two tops of the pillbos point in opposite directions. the vertical compoent of$\vec{D}$ on the two sides must be the same. Thus it is the perpendicuar component of D that is continuous, and the parallel compoent of E that is continuous.

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b) We have two parallel metal plates, area A and separation $d << \sqrt{A}$– much closer together than their lateral dimensions, with a charge of $\pm Q$ on each plate. What is the voltage between the plates, Now insert ultra pure water between the place (relative permativity or dielectric constant ϵ/ϵ_0 of about 80). What will the ratio of charge to voltage be for this new configuration?

Consider a charge Q spread over the top plate and -Q over the bottom plate. Since the plates are metal, the charge per unit area will be Q/A and the E field just outside the plate will be $\sigma Q/\epsilon_0 A$. Now it is the D field that is continuous, and the D field is ϵE so the vertical (assuming the pales lie horizontally) component of E will be $\frac{1}{\epsilon_0}Q/A$ with D being Q/A Since D is continous, it will also be Q/A inside the water, but $E = D/80\epsilon_0 = QA/80\epsilon_0$. The potential is the itnergral of E from one plate to the otehr, and since the air gap is supposed to be tiny, we get

$$V = Ed = Qd/A\epsilon_0 \tag{5}$$

Thus for the same Voltage, the system will hold 80 times as much charge.

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3)[6] a) Consider a region where J equals zero. Show that you can introduce a potential $\Psi(x)$ so that $\vec{B} = -\nabla\Psi$. Recall how we showed that in the discussion of the Aharonov Bohm effect we showed that if we had a region where $\vec{B}$ was zero, we could find a gauge to make $\vec{A}$ equal to 0.

b)

The equaiton for $\vec{H}$ is $\nabla\vec{H} = 0$. Thus just as for the AB we can define a potential

$$\psi(x) = \int_{x_0}^{x} \vec{H} \cdot d\vec{l} \tag{6}$$

This value of ψ is independent of the path chosen from x_0 to x since the difference between the two value along the two paths will equal the curl of H over the surface between the paths. But the curl of H is zero. Then, just as in the gauge transformation for $\vec{A}$, we hare

$$\vec{H} = -\nabla\psi \tag{7}$$

However, in regions where μ is constant, we also have

$$0 = \nabla \cdot \vec{B} = \nabla\mu\vec{H} = \mu\nabla \cdot H \tag{8}$$

saying the divergence of H is zero. Now our construction says that

$$H + \vec{\nabla}\psi(x) = 0 \tag{9}$$

so if $\nabla H = 0$ so is $\nabla^2\psi$. Ie, this gives a potential for H within any region where μ is constant. At the boundaries where μ changes, the above potential will have a delta function boundary term. Or rather one has to take two potentials in the different regions and use the H parallel continuous and B perpendicular continous to get

Fpr part a), $\mu = \mu_0$ everywhere giving a potential for both H and B everywhere.

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