Physics 301
Midterm Exam
Feb 28 2021

This exam is **50 min** in length. It is closed book, but one 8.5x11 inch sheet of paper is allowed. No computers, phones, calculators are allowed

This exam has **4 (four) questions**, all of equal value.

One sheet (8.5x11in) both sides of formulas is allowed).

Redo

When the exam has been marked, and handed back to you, you will have the option of redoing any of the questions on which you did not get full marks and handing the result in for remarking. As your midterm mark you will get the average of the mark you received on the midterm writing itself and the mark you get on the redo of the question, but in no case will you get less than that the mark you received on the midterm marking itself. This ”re-do” will be due 1 week after the results of the marking of the original midterm are sent back to you, with no extension of that time. You may discuss this redo with others, and use any notes, text-books, etc. in doing so, except you may not simply copy someone else’s solution(s), or feed them into any computer programs like, but not limited to ChatGPT to answer it for you.

Over

1. Which of the following are tensor equations. Explain why you say whether they are or not. Each individual lettered term on the RHS is assumed to be a tensor. ϵ_{ijk} is the completely anti-symmetric tensor such that $\epsilon_{123} = \sqrt{g}$. $g = det(g_{ij})$, $e^{ijk} = \sqrt{g}\epsilon^{ijk}$

a)

$$S_i{}^j = W_j Q^{kl} W_l + 5S^l (Q_{il} - Q_{li}) T^j$$

b)

$$S^i = \epsilon^{ijk} \partial_i A_k$$

c)

$$S^i = e^{ijk} \partial_i A_k$$

d)

$$F = \partial_i A^i$$

e)

$$S_{ab} = W_a + R_b$$

2)The surfaces $x = 0,\ y = 0,\ z = 0$ are all conductors. A charge Q is placed at the point $x = y = z = 1$. Where are the image charges located and what are each of their values?

3. We have a charge density distribution on the surface of a sphere of radius R given by

$$\rho(r, \theta, \phi) = \sigma \delta(r - R) \cos(\theta) \tag{1}$$

a) What do you expect the θ, ϕ dependence of the potential for this charge distribution to be?

b)What do you expect the radial dependence of the potential Φ to be both for $r < R$ and for $r > R$. (you do not need to solve the equation exactly– just give arguments as to what you expect the answer to be

4.A conductor has an interior electric field is equal to 0 because an electric field would push the charges around until the electric field is zero inside (but not on the surface). Show that the external electric field must be perpendicular to the surface just at the surface. What is the relation between the surface electric field and the surface charge density in the conductor?

ℓ = 0

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

ℓ = 1

$$
\begin{aligned}
Y_1^{-1}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta &&= \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x-iy)}{r} \\
Y_1^{0}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta &&= \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\frac{z}{r} \\
Y_1^{1}(\theta,\varphi) &= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta &&= -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x+iy)}{r}
\end{aligned}
$$

ℓ = 2

$$
\begin{aligned}
Y_2^{-2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta &&= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)^2}{r^2} \\
Y_2^{-1}(\theta,\varphi) &= \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot\cos\theta &&= \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)\cdot z}{r^2} \\
Y_2^{0}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot(3\cos^2\theta-1) &&= \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot\frac{(3z^2-r^2)}{r^2} \\
Y_2^{1}(\theta,\varphi) &= -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot\cos\theta &&= -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)\cdot z}{r^2} \\
Y_2^{2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{2i\varphi}\cdot\sin^2\theta &&= \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)^2}{r^2}
\end{aligned}
$$

ℓ = 3

$$
\begin{aligned}
Y_3^{-3}(\theta,\varphi) &= \frac{1}{8}\sqrt{\frac{35}{\pi}}\cdot e^{-3i\varphi}\cdot\sin^3\theta &&= \frac{1}{8}\sqrt{\frac{35}{\pi}}\cdot\frac{(x-iy)^3}{r^3} \\
Y_3^{-2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta\cdot\cos\theta &&= \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot\frac{(x-iy)^2\cdot z}{r^3} \\
Y_3^{-1}(\theta,\varphi) &= \frac{1}{8}\sqrt{\frac{21}{\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot(5\cos^2\theta-1) &&= \frac{1}{8}\sqrt{\frac{21}{\pi}}\cdot\frac{(x-iy)\cdot(5z^2-r^2)}{r^3} \\
Y_3^{0}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{7}{\pi}}\cdot(5\cos^3\theta-3\cos\theta) &&= \frac{1}{4}\sqrt{\frac{7}{\pi}}\cdot\frac{(5z^3-3zr^2)}{r^3} \\
Y_3^{1}(\theta,\varphi) &= -\frac{1}{8}\sqrt{\frac{21}{\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot(5\cos^2\theta-1) &&= \frac{-1}{8}\sqrt{\frac{21}{\pi}}\cdot\frac{(x+iy)\cdot(5z^2-r^2)}{r^3} \\
Y_3^{2}(\theta,\varphi) &= \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot e^{2i\varphi}\cdot\sin^2\theta\cdot\cos\theta &&= \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot\frac{(x+iy)^2\cdot z}{r^3} \\
Y_3^{3}(\theta,\varphi) &= \frac{1}{8}\sqrt{\frac{35}{\pi}}\cdot e^{3i\varphi}\cdot\sin^3\theta &&= \frac{-1}{8}\sqrt{\frac{35}{\pi}}\cdot\frac{(x+iy)^3}{r^3}
\end{aligned}
$$

Figure 1: Figure ylm. From https://en.wikipedia.org/wiki/Table_of_spherical_harmonics with signs altered to make $Y_{lm}^* = Y_{l,-m}$.