Physics 530-23
Tutorial 1

1. Consider a tensor in 3-D $h_{ij} = \delta_{ij}$, and a coordinate transformation (where $\{x^1, x^2, x^3\} \equiv \{x, y, z\}$. Consider the coordinate transformation

$$\tilde{x} = x\cos(\theta) + y\sin(\theta); \quad \tilde{y} = y\cos(\theta) - x\sin(\theta); \quad \tilde{z} = z$$

where θ is a constant.Consider the following tensors

$$T^i = \{1, 2, 1\}$$
$$W_i = \{2, 1, 2\}$$
$$H^i{}_j = H^1{}_1 = 1; \quad H^2{}_1 = -1; \quad \text{all others components } 0$$

What are the components in the tilde coordinate system.

2. Given that the metric is

$$ds^2 = dr^2 + r^2 d\phi_d^2 z^2. \tag{1}$$

and the coordinates are $\{x^1, x^2, x^3\} \equiv \{r, \phi, z\}$, what is the inverse metric, the determinant of the metric, and the component of the anti-symmetric tensor ϵ^{123} Take $A_i = \{0, r^2, 0\}$ for $r < 1$ and is $\{0, 1, 0\}$ for $r > 1$, what are the components of B^i where $B^i = \epsilon^{ijk}\partial_j A_k$.

3.Given the metric $ds^2 = dx^2 + dxdy + dy^2 + dz^2$ what are the components of the metric, the inverse metric and the determinant of the metric? $\{x^1, x^2, x^3\} \equiv \{x, y, z\}$