Physics 530-23
Tutorial 2

1. Given that the metric is

$$ds^2 = dr^2 + r^2 d\phi^2 + dz^2. \tag{1}$$

and the coordinates are $\{x^1, x^2, x^3\} \equiv \{r, \phi, z\}$, what is the inverse metric, the determinant of the metric, and the component of the anti-symmetric tensor ϵ^{123} Take $A_i = \{0, r^2, 0\}$ for $r < 1$ and is $\{0, 1, 0\}$ for $r > 1$, what are the components of B^i where $B^i = \epsilon^{ijk}\partial_j A_k$.

2.Given the metric $ds^2 = dx^2 + dxdy + dy^2 + dz^2$ what are the components of the metric, the inverse metric and the determinant of the metric? $\{x^1, x^2, x^3\} \equiv \{x, y, z\}$

B field in a vortex core looks approximately likes

$$B_z \approx B_0(1 - A^2 r^2) \quad r < 1/A; \qquad 0 \quad r > 1/A \tag{2}$$

with all other components are 0. Calculate the curl of this magnetic field. What must be the current in the superconductor to create this magnetic field.

Do it both in cylindrical polar coordinates z, r, θ whose metric is $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$ and in cartesian coordinates x, y, z, where the metric is $ds^2 = dx^2 + dy^2 + dz^2$, and $x = r\cos(\theta), \quad y = r\sin(\theta)$ or $r^2 = (x^2 + y^2)$ Recall that curl B is the $\mu_0 J$.

Which coordinate system is easier?