1, 2.

$A_1, \tilde{A}_1 \quad B_2, \tilde{B}_2$

$A_1, \tilde{A}_1 \quad B_2, \tilde{B}_2$

Entangled if

$\exists\, A_1, B_1$

$\langle A_1 B_1 \rangle \neq \langle A_1 \rangle \langle B_1 \rangle$

If there exist.

Quantum state is entangled.

$$C = A_1 B_2 + A_1 \tilde{B}_2 + \tilde{A}_1 B_2 - \tilde{A}_2 \tilde{B}_2$$

$$C^2 = A_1^2 B_2^2 + A_1^2 \tilde{B}_2^2 + \tilde{A}_1^2 B_2^2 + \tilde{A}_1^2 \tilde{B}_2^2$$

$$+ [A_1, \tilde{A}_1][B_2, \tilde{B}_2]$$

Lets choose $A, \tilde{A}, B, \tilde{B}$ to have only $\pm$ as e-values.

$$C^2 = \underline{4} + [A_1, \tilde{A}_1][B_2, \tilde{B}_2]$$

$$[A, \tilde{A}]^{\dagger} = [\tilde{A}^{\dagger}, A^{\dagger}]$$

$$= [\tilde{A}, A] = -[A, \tilde{A}]$$

$= i$ Herm op.

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad [\sigma_1, \sigma_2 \cdot \sigma_2 \sigma_1] = i2\sigma_3$$

$$A_1 = \sigma_1, \quad \tilde{A}_1 = \sigma_2, \quad B = \Sigma_1, \quad \tilde{B} = \Sigma_2$$

$$c^2 = 4 - 4\,\underline{\sigma_3 \Sigma_3}$$

$$c^2 = 0,\ 8$$

$$c \to 0,\ \pm 2\sqrt{2}$$

$$(\sigma_3, \Sigma_3) \Rightarrow (1, -1), \quad \sigma_3, \Sigma_3 (-1, 1)$$

$$|1, -1\rangle \qquad |-1, 1\rangle$$

$$\alpha|1, -1\rangle + \beta|-1, 1\rangle \Rightarrow \alpha = \pm\beta$$

$$c = \pm 2\sqrt{2}$$

In Q.M we can find states such that

$$-2\sqrt{2} \leq \langle C \rangle \leq 2\sqrt{2}$$

For classical system.

$$-2 \leq \langle C \rangle \leq 2$$

Quantum correlations are (can be) stronger than any classical system

Entanglement depends on how you divide a system into two subsystems

$$H = \frac{1}{2}\left(P_1^2 + P_2^2 + X_1^2 + X_2^2 + 2\delta(t) X_1 X_2\right)$$

start with ground state of each H.O.

$|0\rangle|0\rangle \qquad \langle A B\rangle = \langle A\rangle\langle B\rangle$