$|\psi\rangle$, A , A $\quad A|a\rangle = a|a\rangle$

$$|\langle a|\psi\rangle|^2 = P(a)$$

$|\psi_n\rangle$ Choose P_n

$$P_1 |\langle a|\psi_1\rangle|^2 + P_2 |\langle a|\psi_2\rangle|^2 + \dots$$

$$\langle A\rangle = \sum_n P_n \langle\psi_n|A|\psi_n\rangle$$

$$Tr = \sum_n \langle\phi_n|B|\phi_n\rangle$$

$|\phi_n\rangle$ complete on set of states.

$$B = |\psi_1\rangle\langle\psi_2|$$

$$B|\psi\rangle = |\psi_1\rangle\langle\psi_2|\psi\rangle$$

$$\mathrm{Tr}\, B = \langle \psi_2 | \psi_1 \rangle$$

$$\sum_n \langle \phi_n | \psi \rangle | \phi_n \rangle = | \psi \rangle$$

$$\rho = \sum_n p_n | \psi_n \rangle \langle \psi_n |$$

$$\mathrm{Tr}(\rho A) = \sum_n p_n \langle \psi_n | A | \psi_n \rangle$$

$$= \langle A \rangle$$

$$\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$$

$$\mathrm{Tr}\,[X, P] \neq 0 \qquad = i\,\mathrm{Tr}\, I = \infty$$

$$\mathrm{Tr} = \sum_m \langle \phi_m | A | \phi_m \rangle$$

$$\stackrel{?}{=} \sum_m \langle \tilde{\phi}_m | A | \tilde{\phi}_m \rangle \quad ?$$

$$| \tilde{\phi}_n \rangle = U | \phi_n \rangle$$

$$= \sum_m \langle \phi_m | U^{+} A U | \phi_m \rangle$$

$$= \mathrm{Tr}\ U^{+} A U = \mathrm{Tr}\ A \underbrace{U^{+} U}_{I}$$

$$= \mathrm{Tr}\ A$$

ρ is playing role of the state in expectation values.

System of two parts.
Look at operators on the one of parts
state of whole

$$|\Psi\rangle = \sum_m |\psi_n\rangle_1 |\Phi_n\rangle_2$$

$A_1 \quad \langle A\rangle = \sum_{m} \langle\Phi_m|_2 \langle\psi_n|_1 A |\psi_m\rangle_1 |\Phi_m\rangle_2$

$$= \sum_{m} \langle\Phi_n|_2 |\Phi_m\rangle_2 \langle\psi_n|A|\psi_m\rangle_1$$

$$\rho_1 = \sum_{m} \langle \Psi_m | \Psi_m \rangle_2 \, |\psi_m\rangle_1 \langle \psi_m|_1$$

Hermitian $\rho^\dagger = \rho$.

Reduced Density matrix

$$Tr(\rho_1) = \sum_{m} \langle \Psi_m | \Psi_m \rangle \langle \psi_m | \psi_m \rangle$$

$$\langle \psi | \psi \rangle = 1.$$

$=$

Eigenvalues are all positive.

$|\lambda\rangle, \lambda \rightarrow$ eigenstates of ρ

$$\Longrightarrow \sum_{i} \lambda_i = 1, \quad \lambda_i \text{ are like probabilities}$$

The reduced density matrix
is exactly like the
"prob choice" of states.

So a pure state for system
is like a prob. choice for
the subsystem.

Any Density matrix, one can
model it by a pure state for
that system plus another
system.

Lets say we have system has a pure state

$$\rho_1 = \sum_m \langle \tilde{\Psi}_m | \tilde{\Psi}_m \rangle_2 \underline{|\Psi_m\rangle_1 \langle \Psi_m|_1}$$

If we choose $|\Psi_m\rangle$ to be the eigenstates of ρ

$$\rho_1 = \sum_m \lambda_m \underline{|\Psi_m\rangle_1 \langle \Psi|_m}$$

$$\langle \tilde{\Psi}_n |_2 |\tilde{\Psi}_m\rangle_2 = \delta_{nm} \lambda_n$$

Thus the expans factors for system 2 must also be an orthonormal system of states and density matrix

$$\rho_2 = \sum \lambda_n |\tilde{\Psi}_n\rangle\langle\tilde{\Psi}_n|$$

→ eigenvalues of ρ_2 are same as e-values of ρ_1

→ Rank of ρ_1 = # e-values which are non zero

= Rank of ρ_2

If Rank $\rho_1 = 1$

then state is product state

$|\psi\rangle = |\psi_1\rangle_1 |\tilde{\psi}_1\rangle_2$

$\langle\psi| A \cdot B |\psi\rangle = \langle\psi_1|A|\psi_1\rangle \langle\tilde{\psi}_1|B|\tilde{\psi}_1\rangle$

No correlations.

Rank of $\rho \Rightarrow$ measure of correlations.

Popular measure: Entropy.

stat mech

$$S = \sum_i -p_i \ln p_i$$

Von Neumann entropy ⟹ λ_i entries of density matrix.

$$\sum_i -\lambda_i \ln \lambda_i \qquad \rho = \text{density matrix}$$

If $p_i = \frac{1}{D}$ where D is dimension of Hilbert space,

$$S = \sum_{n=1}^{D} -\frac{1}{D} \ln \frac{1}{D} = \ln D$$

Note that the statement that the e-values of ρ_1, ρ_2 are same is true only for pure states.

For example:

$$\rho = \frac{1}{2}\left(|1\rangle_1 |-1\rangle_2 \langle -1|_2 \langle 1|_1 + |1\rangle_1 |+1\rangle_2 \langle +1|_2 \langle 1|_1\right)$$

$$\rho_1 = \cancel{\tfrac{1}{2}}\left(|1\rangle\langle 1|\right)$$

$$\rho_2 = \left(\tfrac{1}{2}|-1\rangle\langle -1| + \tfrac{1}{2}|1\rangle\langle 1|\right)$$

Decoherence

Pure state for $\underbrace{\text{bipartite system}}_{\text{2 subsystems.}}$

Product state

$$|\psi\rangle_1 \underbrace{|\tilde{\Psi}\rangle_2}$$

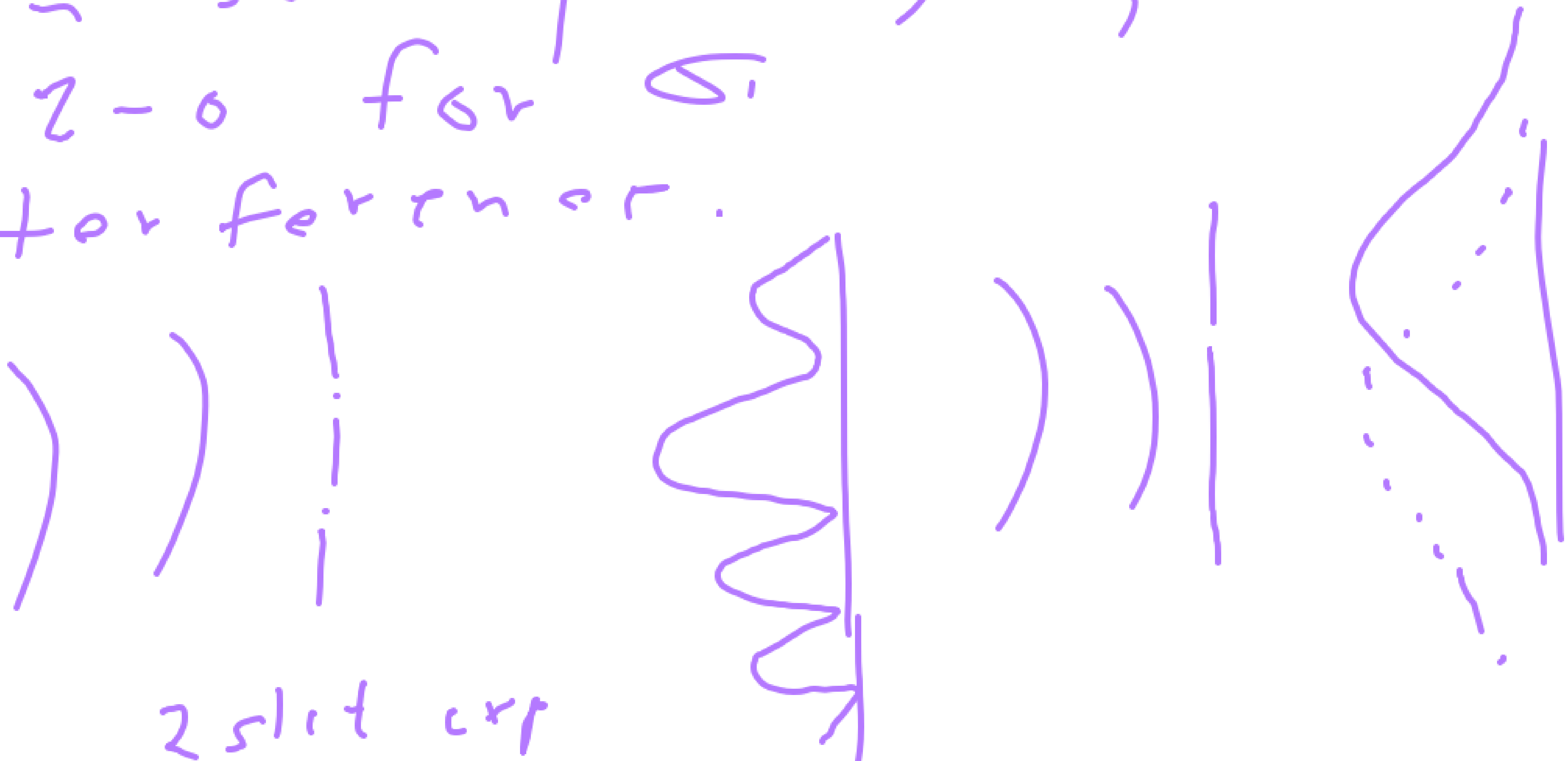
2 level system. $|\psi_1\rangle$ were a state of σ_3, $|\uparrow\rangle \Longrightarrow \sigma_1 \Longrightarrow$ 50-50 prob of getting +1 or -1.

but we would get 100% prob of getting +1 for σ_3.

If started with $\frac{|1\rangle + |-1\rangle}{\sqrt{2}}$

then 50-50 for σ_3, σ_2, and 100%-0 for σ_1

Interference.



2 slit exp

Interaction which creates an entangled state

$$|\uparrow\rangle_2 \otimes \left(\frac{|1\rangle_1 + |-1\rangle_1}{\sqrt{2}}\right)|1\rangle_2$$

$\Downarrow$ Interaction

$$\left(\frac{|1\rangle_1|1\rangle_2 + |-1\rangle_1|-1\rangle_2}{\sqrt{2}}\right) \sigma_{z_1}\sigma_{z_2}$$

Density matrix for system 1

$$\rho_1 = \frac{1}{2}|1\rangle_1\langle 1|_1 + \frac{1}{2}|-1\rangle_1\langle -1|_1$$

$$= \frac{1}{2} I$$

For $\frac{1}{2}I$ the prob of getting $+1$ for any of $\sigma_1, \sigma_2, \sigma_3$

$$= \frac{1}{2}$$

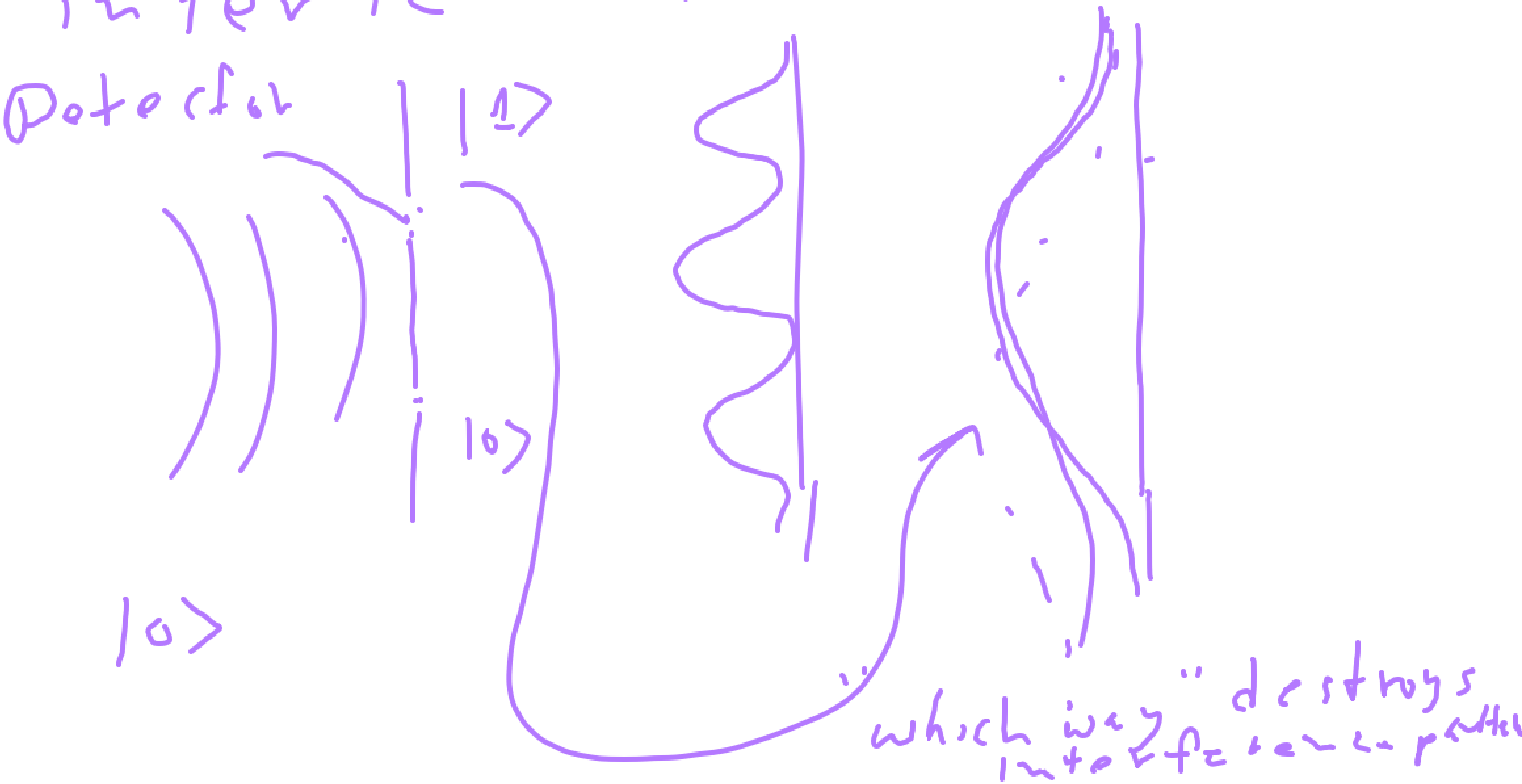
$$I = \rightarrow |1\rangle\langle 1| + |-1\rangle\langle -1|$$

$$= \left(\frac{1}{\sqrt{2}}\left(|1\rangle + |-1\rangle\right)\left(\langle 1| + \langle -1|\right)\right.$$

$$\left. + \frac{1}{\sqrt{2}}\left(|1\rangle - |-1\rangle\right)\left(\langle 1| - \langle -1|\right)\right)$$

$\Rightarrow$ No interference.

Correlations destroy interference.



Quantum Eraser