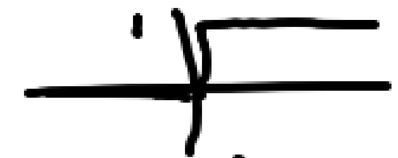
Weak Meas.

Meas. apparatus. $H_f = 0$

$P, \underline{q}$.

System: S Measure at $t = 0$



Heaviside step func.

$S.p$ e-val s $|s\rangle$

$$H_I = \epsilon\, \delta(t)\, S\, p$$

$$q = q_0 + \Theta(t)\, \epsilon\, S(t=0)$$

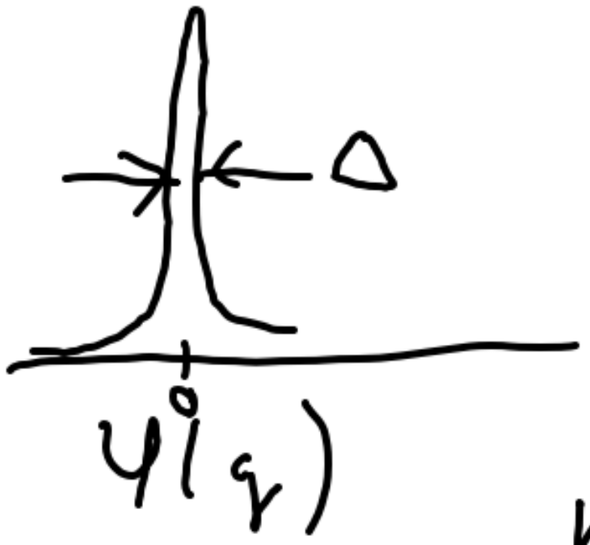
Heis: $i \partial_t q = i \epsilon\, \delta(t)\, S$

$\psi(q) \Longrightarrow \psi(q - \epsilon s)$ for $t > 0$

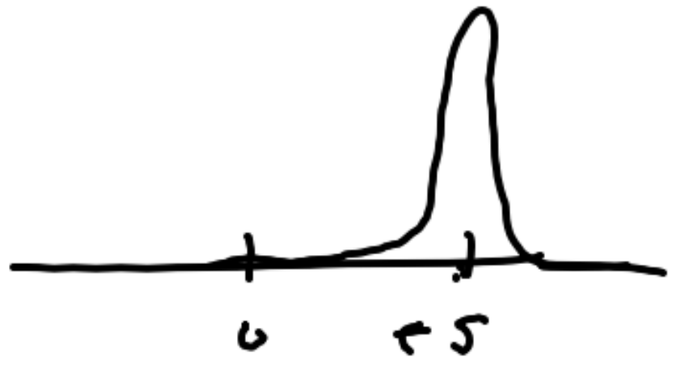
State of S is $|s\rangle$

$$U\left(\psi(q) \sum \alpha_s |s\rangle\right) = \sum_s \alpha_s U\left(\psi(q)|s\rangle\right) \quad \text{Entangl.}$$

$$= \sum_s \alpha_s \underbrace{\psi(q - \epsilon s)\, |s\rangle}$$



$\Rightarrow$



Measure $q \Rightarrow$ value meas $\in s \pm \epsilon\Delta$

Uncert comes from initial uncert of meas. apparatus.

After meas.:

$$\sum_s \alpha_s \psi(q - \epsilon s) \,|\, s\rangle \approx \alpha_{s_0} |\, s_0\rangle$$

$$= \alpha_{\frac{q}{\epsilon}} \left|\left[\frac{q}{\epsilon}\right]\right\rangle$$

If Δ is large.

Broad $\psi(q) \Rightarrow$ uncertain s

$\psi(q) \sum \alpha_s |s\rangle \Rightarrow \sum \psi(q - \epsilon s) \alpha_s |s\rangle$

$\psi(q - \epsilon s)$ damps out components of s far from q

Weak meas. $\Rightarrow$ leaves state alone ϵ near measured value of s

2-time meas

Set conditions at 2 times.
Measure at 1st time
Weak meas. at intermed. time

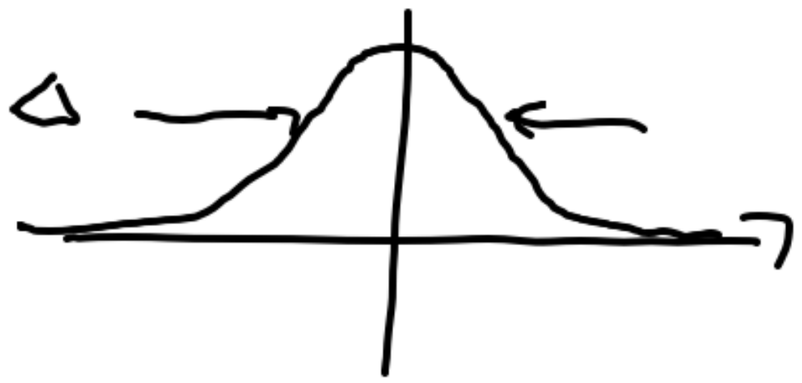
Specific instance.
2 level system, $\sigma_1, \sigma_2, \sigma_3$

9am $\sigma_3 \to +1$ 10am $\frac{1}{\sqrt{2}}(\sigma_1 + \sigma_3) = S$
11am $\sigma_1 \to +1$ e.val ± 1

what value do I measure?

$$\psi(q) = \frac{1}{\sqrt{\sqrt{2\pi}\,\Delta}}\, e^{-q^2/4\Delta^2}$$

$$\int |\psi(q)|^2 q^2\, dq = \Delta^2$$



S, $s = \pm 1$, $\pm\frac{1}{2}$

$|+\rangle, |-\rangle \rightarrow$ eigenstates of σ_3

$S \rightarrow$ spin in 45° direction

$$\cos\tfrac{\pi}{8}|+\rangle + \sin\tfrac{\pi}{8}|-\rangle,$$

$$\cos\tfrac{\pi}{8}|-\rangle - \sin\tfrac{\pi}{8}|+\rangle$$

$|0\rangle$ $\quad$ $|1\rangle$

$\psi(q)|+\rangle$ Sestudes $|0\rangle, |1\rangle$

$\psi(q)\left(\cos\frac{\pi}{4}|1\rangle - \sin\frac{\pi}{4}|0\rangle\right)$

$\Rightarrow \psi(q-\epsilon)\cos\frac{\pi}{8}|1\rangle - \sin\frac{\pi}{8}\psi(q+1)|0\rangle$

Make final meas. $\frac{1}{\sqrt{2}}(\sigma_x \pm \sigma_y)$ $\rightarrow$ estimate

σ_y are $\frac{\pi}{4}$ away from 0 $\Rightarrow \frac{\pi}{8}$ away

from $\frac{\pi}{8}$

$\cos\theta\sigma_y + \sin\theta\sigma_x \Rightarrow \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$

$\cos\frac{\theta}{2}|-\rangle - \sin\frac{\theta}{2}|+\rangle$

$$\psi(q-\epsilon)\cos\frac{\pi}{\beta}|\uparrow\rangle - \sin\frac{\pi}{\beta}|0\rangle\,\psi(q+\epsilon)$$

$$\left(\cos\frac{\pi}{\beta}|\uparrow\rangle - \sin\frac{\pi}{\beta}|\downarrow\rangle\right) \quad \cos\frac{\pi}{\beta}|\downarrow\rangle + \sin\frac{\pi}{\beta}|\uparrow\rangle$$

$\rightarrow$ I now need to select only those terms which are $|\uparrow\rangle$

$$\psi(q-\epsilon)\left(\cos\frac{\pi}{\beta}\right)^2 - \left(\sin\frac{\pi}{\beta}\right)^2\psi(q+\epsilon)$$

$$P(q) = \frac{|\quad|^2}{\int |\quad|^2\, dq'}$$

If $\sigma \gg 1$, then 2 terms tend to cancel.

$$\psi(q-\epsilon) + \psi(q+\epsilon)$$

overlap.

Expand in ϵ to 1st order.

$$\underbrace{\left(\cos\frac{\pi}{8}^2 - \left(\sin\frac{\pi}{8}\right)^2\right)}_{\cos\frac{\pi}{4}} \psi(q) - \underbrace{\left(\cos\frac{\pi}{8}^2 + \sin\pi/8^2\right)}_{1} \epsilon\, \partial_q \psi$$

$$\approx \cos\frac{\pi}{4}\, \psi\left(q - \frac{\epsilon}{\cos\pi/4}\right)$$

*

Measured spin value is

$$\frac{1}{\cos \pi/4} = \sqrt{2}$$

Measure spin is 1.4 x max value

Generic.

For large Δ the mean displacement of pointer.

$$\frac{\langle f|S|i\rangle}{\langle f|i\rangle}$$

Assume $\psi(q)$ smooth so Taylor series is good approx. and Δ "large"