Final Exam
General Relativity, Physics 530
Apr. 2025

This exam consists of 6 questions on 3 pages. You are to do all 5 questions.

You are not to communicate with anyone else about, or show anyone else the exam or parts of it during the exam time.

You are not to use ChatGPT or any other AI during the course of the exam. HI (your own Human Intelligence) is of course allowed.

The exam is ”open book”.

This exam is a 15 Hr exam.

It ends at 11:59PM. Please send the finished exam to
unruh@physics.ubc.ca
My phone number in case of questions etc is
778 238 7962

1. Consider the metric

$$ds^2 = dt^2 - \frac{rdr^2}{r-2M} - r^2\left((d\theta)^2 + sin(\theta)^2 s\phi^2\right); \qquad c = 1 \tag{1}$$

This is essentially the metric used in the movie Interstellar for the wormhole out near Saturn's orbit.

a) Find the geodesics of this metric for both massive and massless particles. Find the circular obits.

b) Consider two radial orbits at $theta = 0$ and $\theta = \epsilon << 1$ and $\phi = 0$. released from $10M$ at the same time, $t = 0$, with radial inward velocity v when released. What is the distance between the two orbits at any time t to lowest order in ϵ. What is the maximum acceleration of that distance, if the Schwartzschild radius (ie, $2M$) is 2Km?

c) If the spaceship has a width of 50m, and the velocity is 1km in 20 sec. (which is an estimate from the movie of how long the spaceship took to fly throught the throat of the wormhole). The area density of the walls is 0.1 metric tonne per square meter, what is the force per square meter, on the walls to keep the spaceship the same size throughout. Do you think the spaceship would have survived?

Note that it might or might not be easier to solve this in Flamm coordinates.

2) Consider the metric

$$ds^2 = H(u, x, y)du^2 + 2dudv - dx^2 - dy^2 \tag{2}$$

a) Find the components of the inverse metric g^{ij}

b) Find the determinant of the metric, and show that it is a constant.

c) show that this metric obeys the harmonic condition, $\partial_i(\sqrt{|g|}g^{ij}) = 0$ (This is called the Harmonic condition, since, if each of the coordinates obeys the massless scalar field equation, then the metric must obey that condition.)

d) Find the non-zero Christofel symbols. In particular show that $\Gamma^u_{ij} = 0$ and $\Gamma^i_{vk} = 0$ and that only Γ^x_{uu} and Γ^y_{uu} are non-zero for Γ^x_{ij} and Γ^y_{ij} terms. Thus show that the $\Gamma\Gamma$ terms in R_{ij} are zero.

Note that

$$\Gamma^k ik = \frac{1}{\sqrt{|g|}}\partial_i\sqrt{|g|} \tag{3}$$

where $g = \text{determinant}(g_{ij})$ (See problem 2 below).

e)Show that Einstein's equations $G_{ij} = 0$ are linear in H, and find the equation that H must solve.

v is a null coordinate (the $x,\ y,\ u$ constant trajectories are null trajectories)

3. Consider the Hamiltonian

$$H = \frac{1}{2}(p^2 - q^2) \tag{4}$$

a) If $p_1(t),\ q_1(t)$ and $p_2(t),\ q_2(t)$ are two linearly independent real solutions of the Hamiltonian equations, find the the norm of the complex solution $\tilde{p}(t) = p_1 + ip_2,\ \ \tilde{q}(t) = q_1 + iq_2$

b) Find two linearly independent solutions for the Hamiltonian equations with

$$p_1(0) = 1;\ \ q_1(0) = 0 \tag{5}$$
$$p_2(0) = 0;\ \ q_2(0) = 1 \tag{6}$$

Find the norm for these two modes and find a (complex) mode with unit norm. Show explicitly that the norm is indendent of time. What are the annihilation (A) and creation $(A^\dagger)$ operators corresponding to this˜mode in terms of P_0 and Q_0 the initial quantum momentum and position operators? What are P_0 and Q_0 in terms of A and $A^\dagger$?

c) What is the expectation value for the energy $\frac{1}{2}(P^2 - Q^2))$ where P is operator corresponsing to the momentum in the vacuum state for your modes.

4. Consider the metric (called Schwarzschild-De Sitter)

$$ds^2 = \left(1 - \frac{2M}{r} - \left(\frac{r}{r_0}\right)^2\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r} - \left(\frac{r}{r_0}\right)^2} - r^2(d\theta^2 + \sin(\theta)^2 d\phi^2) \quad (7)$$

Find the circular ($r = R$ aconst, $\theta = 0$) timelike and null geodesic orbits in this metric. What sets the limits of r for such an orbit to exist.

What are the conditions on r that a timelike circular geodesic orbit is possible? Is there a maximum value of r for which circular orbits are possible?

5 The last 50 years have been an amazing flowering of tests of General Relativity, a theory most thought was incapable of tests.

Describe at least three of the classical tests (pre 1950) of General relativity.

Describe the experimental detection of gravitational waves, how it was accomplished some of the noise sources which had to be overcome.

6. Make up a question on some topic in the physics of General Relativity not covered by any of the above questions on this exam, and answer the question.

You will be marked both on the question and on the answer. A trivial question with trivial answer will receive few mark while a very difficult but interesting question with a poor answer will receive at least half marks. A simple question with a complete answer would receive good marks.

The purpose of this question is for you to tell me something that you know or have learned from the course, without the straightjacket of my own questions. (Note as a standard that none of the questions on this exam are, by definition, trivial, although some would receive fewer marks than others for the question itself–i.e., some are straightforward questions while others demand some lateral thinking on the part of the student.)