Physics 530-11
Penrose diagram of Schwartzscild

0.1 Kruscal Schwartzschild

The Schwartzschild metric is

$$ds^2 = (1-\frac{2M}{r})dvdu - r^2 d\Omega^2 \tag{1}$$

where $d\Omega^2 = d\theta^2 + sin(\theta)^2 d\phi^2$ and

$$u = t - r^* \tag{2}$$
$$v - t + r^* \tag{3}$$
$$r^* = r + 2 * M\ln(\frac{r}{2M} - 1) \tag{4}$$

Thus$v - u = 2r^* + 4\ln(\frac{r-2M}{2M})$ and $e^{(v-u)/4M} = (\frac{r-2M)}{2M}e^{r/2M}$ and we can write

$$ds^2 = \frac{2M}{r}e^{-r/2M}e^{(v-u)/4M}dudv - ((v-u)/2 - 2Mln(\frac{r}{2M} - 1)) \tag{5}$$

We now define

$$\begin{array}{ll} U = -4Me^{-u/4M} & u > 0 \\ V = 4Me^{v/4M} & v < 0 \end{array} \tag{6}$$

Now define

$$V = tan(\hat{V}/4M) \tag{7}$$
$$U = tan(\hat{U}/4M) \tag{8}$$

For large r, $v-u = 2r^* = 2r(1+2M\frac{\ln(r/2M-1)}{r})$ This approaches r for large r, but the $2M\ln(r/2M-1)$ is a singular function as $r = sin(V)sin(-U)/(cos(V)cos(U))$ goes to infinity. Thus the $U = -\pi/2$ and $V = \pi/2$ surfaces are well behaved but the metric as one approaches them is not very well behaved. and is especially badly behaved as one approaches both of those limits. I_0, spacelike infinity, is badly behaved not only in the conformal factor going to infinity there, but also that the conformal metric is badly behaved there. Similarly, the metric at $I^{\pm}$ is badly behaved unlike the case for flat spacetime.

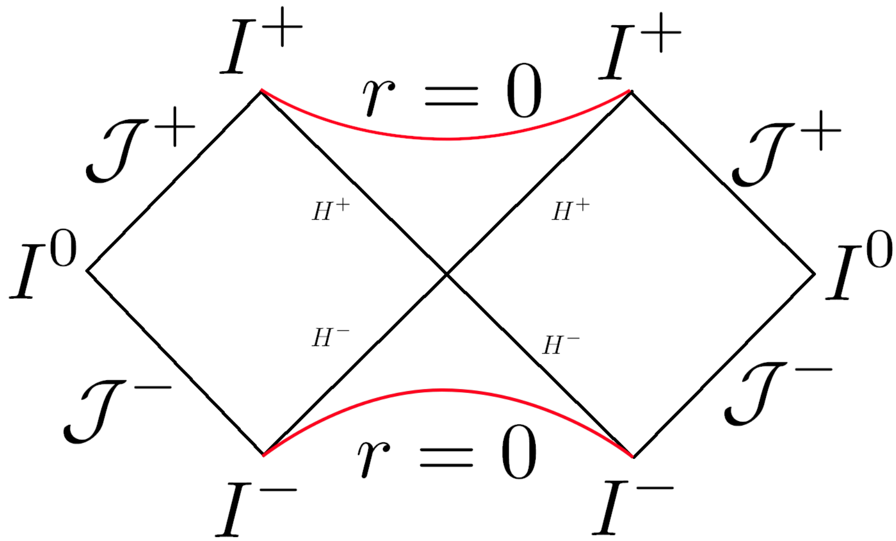

Figure 1: The Penrose conformal diagram for the eternal (Kruscal) black hole.